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TRANSPORT CROSS SECTION
EXPRESSED IN TERMS OF PHASE SHIFT

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TRANSPORT CROSS SECTION EXPRESSED IN TERMS OF PHASE SHIFTS

The differential cross-section for elastic scattering is (see Mott and Massey Ch. II):

$$d\sigma = 2\pi \left| f(\theta) \right|^2 \sin\theta \, d\theta$$

where:

$$f(\theta) = (1/2ik) \sum_{n=0}^{\infty} (2n+1) \left[e^{2i\eta_n} - 1\right] P_n (\cos\theta)$$

This leads to:

$$\left| f(\theta) \right|^2 = (1/4k^2) \sum_n \sum_{n'} (2n+1)(2n'+1) \left[e^{2i\eta_n} - 1\right]\left[e^{-2i\eta_{n'}} - 1\right] P_n P_{n'}$$

We can transform

$$\left[e^{2i\eta_n} - 1\right] \left[e^{-2i\eta_{n'}} - 1\right]$$

$$= e^{i(\eta_n - \eta_{n'})} \left[\left(e^{i\eta_n} - e^{-i\eta_n}\right) \left(e^{-i\eta_{n'}} - e^{i\eta_{n'}}\right)\right]$$

$$= 4 \, e^{i(\eta_n - \eta_{n'})} \sin\eta_n \sin\eta_{n'}$$

which leads to

$$\left| f(\theta) \right|^2 = (1/k^2) \sum\sum (2n+1)(2n'+1) \cos(\eta_n - \eta_{n'}) \sin\eta_n \sin\eta_{n'} P_n P_{n'}$$

From this follows immediately the well-known formula for the total elastic cross section

$$\sigma_t = \int d\sigma = (4\pi/k^2) \sum (2n+1) \sin^2\eta_n$$

Another quantity of interest is the transport part of the elastic cross section

$$\sigma_{tr} = \int (1 - \cos\theta) d\sigma = \sigma_t - \int \cos\theta \, d\sigma$$

We need integrals

$$\int_{-1}^{1} P_n P_{n'} \, u \, du, \text{ when } u = \cos\theta$$

The only non-vanishing integrals are those for which $n' = n \pm 1$

$$\int P_n P_{n+1} u \, du = \frac{2n+2}{(2n+1)(2n+3)}$$

Thus:

$$\sigma_t - \sigma_{tr} = (8\pi/k^2) \Sigma (n+1) \cos(\eta_{n+1} - \eta_n) \sin\eta_{n+1} \sin\eta_n$$