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TRANSPORT CROSS SECTION  
EXPRESSED IN TERMS OF PHASE SHIFT

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# TRANSPORT CROSS SECTION EXPRESSED IN TERMS OF PHASE SHIFTS

The differential cross-section for elastic scattering is (see

Mott and Massey Ch. II):

$$d\sigma = 2\pi \left| f(\theta) \right|^2 \sin \theta d\theta$$

where:

$$f(\theta) = (1/2ik) \sum_{n=0}^{\infty} (2n+1) \left[ e^{2i\eta_n} - 1 \right] P_n(\cos \theta)$$

This leads to:

$$\left| f(\theta) \right|^2 = (1/4k^2) \sum_n \sum_{n'} (2n+1)(2n'+1) \left[ e^{2i\eta_n} - 1 \right] \left[ e^{-2i\eta_{n'}} - 1 \right] P_n P_{n'}$$

We can transform

$$\begin{aligned} & \left[ e^{2i\eta_n} - 1 \right] \left[ e^{-2i\eta_{n'}} - 1 \right] \\ & = e^{i(\eta_n - \eta_{n'})} \left[ \left( e^{i\eta_n} - e^{-i\eta_n} \right) \left( e^{-i\eta_{n'}} - e^{i\eta_{n'}} \right) \right] \\ & = 4 e^{i(\eta_n - \eta_{n'})} \sin \eta_n \sin \eta_{n'} \end{aligned}$$

which leads to

$$\left| f(\theta) \right|^2 = (1/k^2) \sum_n \sum_{n'} (2n+1)(2n'+1) \cos(\eta_n - \eta_{n'}) \sin \eta_n \sin \eta_{n'} P_n P_{n'}$$

From this follows immediately the well-known formula for the total elastic cross section

$$\sigma_t = \int d\sigma = (4\pi/k^2) \sum (2n+1) \sin^2 \eta_n$$

Another quantity of interest is the transport part of the elastic cross section

$$\sigma_{tr} = \int (1 - \cos \theta) d\sigma = \sigma_t - \int \cos \theta d\sigma$$

We need integrals

$$\int_{-1}^1 P_n P_{n'} u du, \text{ when } u = \cos \theta$$

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The only non-vanishing integrals are those for which  $n' = n \pm 1$

$$\int P_n P_{n+1} u \, du = \frac{2n+2}{(2n+1)(2n+3)}$$

Thus:

$$\sigma_t - \sigma_{tr} = (8\pi/k^2) \sum (n+1) \cos(\eta_{n+1} - \eta_n) \sin\eta_{n+1} \sin\eta_n$$

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